THEON OF SMYRNA

ASTRONOMY - ON THE SPHERICAL FORM OF THE EARTH

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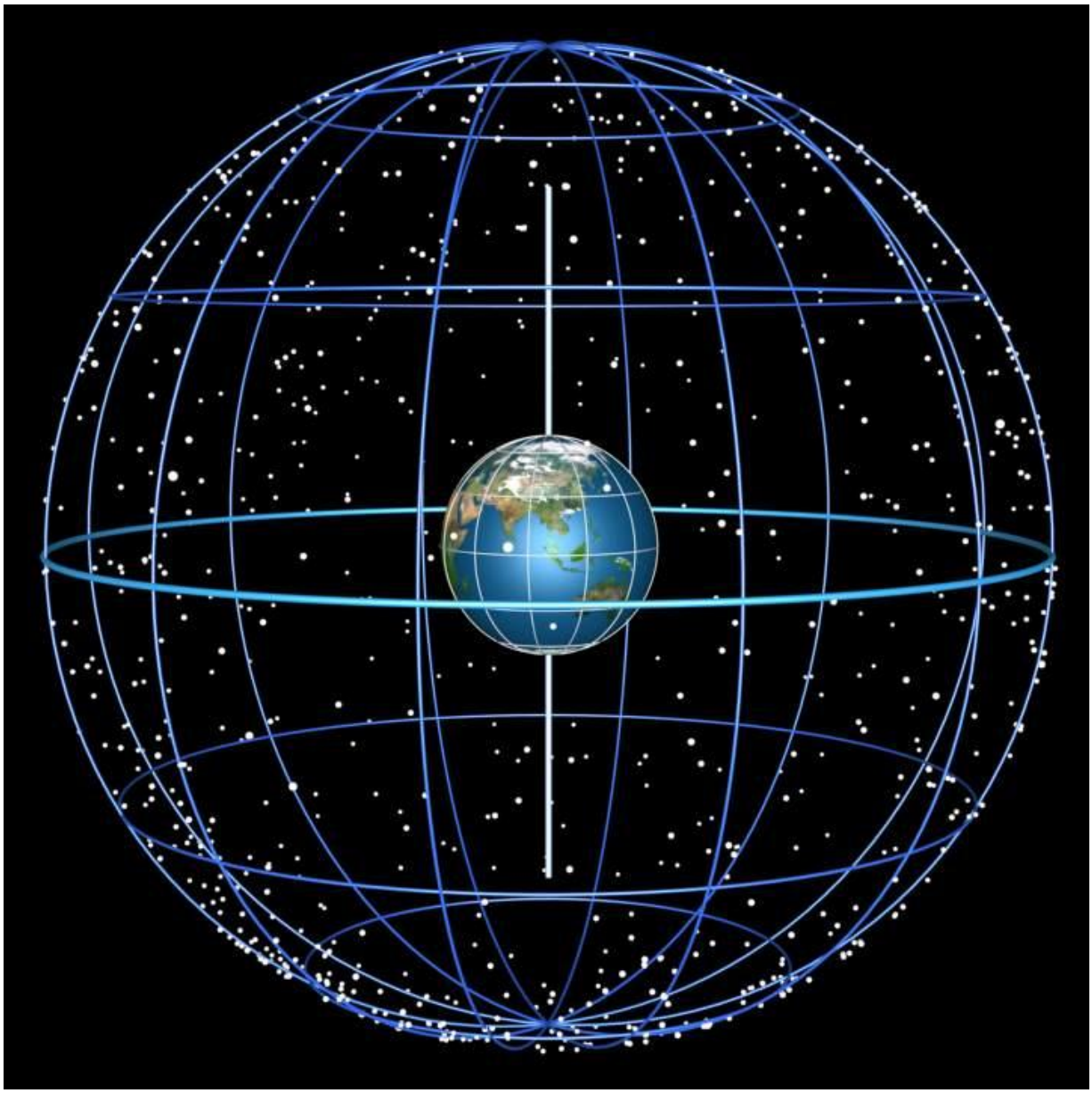
The entire world is a sphere and the earth, which it itself a spheroid, is placed in the middle. That the earth is the center of the universe and that it is but a point in relationship to the size of the universe: this is what must be established before anything else. A precise exposition of this doctrine would require such length consideration of so many writings that it would instead be sufficient to summarize here what we have to say, by recalling the summary notions transmitted to us by Adrastus.



We would say therefore that the world and the earth are spherical and the earth is the center of the world, and that it is only a point in it. This results from the fact that, for inhabitants of the same place, all the celestial bodies rise and set and rise again at the same points, and always accomplish the same revolutions.

The sphericity of the world is again demonstrated by reason of the fact that, from each part of the earth, as far as our senses can tell, half of the sky is seen above us, while we assume the other half to be hidden by the earth and not able to be perceived. Furthermore, if we look at the extreme points of the sky, all the visual rays appear equal to us, and if diametrically opposed stars describe a great circle, one is setting while the other is rising. If the universe, instead of being spherical, were a cone or a cylinder, or a pyramid or any other solid, it would not produce this effect on earth: one of its parts would appear larger, another smaller, and the distances from earth to heaven would appear unequal.

And first of all, the earth is spherical from east lowest, the rising and setting of the same stars certainly prove this. They take place early for the inhabitants of eastern regions, and later for those of the western regions. A single lunar eclipse further shows this: it is produced in the same brief period of time, but for all those seeing it, it appears at different hours of the day. The further eastward one is, the more advanced the hour will be that one sees it, and the sooner one will have seen a greater pan. Because of the curved form of the earth, the sun does not illuminate the whole surface of the earth at the same time, and the shadow that the earth projects moves according to a fixed order, the phenomenon taking place at night.

It is again evident that the earth is convex from north to south: in fact for those going southward, for the measure that they advance, many stars, which are always visible for us in their movement around the pole, have a rising and a setting. In the same way, other stars, always invisible for us in their movement around the pole which is hidden for us, have for them a rising and a setting. Thus the star called Canopus (from the constellation of Argo, one of the most brilliant stars in the northern hemisphere. According to ancient lore, Canopus was the captain of Menelaus' boat - brother of Agamemnon and husband of Helen – when they sailed against Troy) is invisible in lands further north than Cnide (city of Carie, Asia Minor); but it is visible in more southerly lands, and always rises higher and higher to the measure that one is further from the north. On the contrary, when one goes from south towards the north, many of the stars which arc seen rising and selling in the south disappear, entirely, while others, situated in the region of the Bears, and which used to have a rising and a setting, become always visible, and as many more of these are seen as one advances northwards.

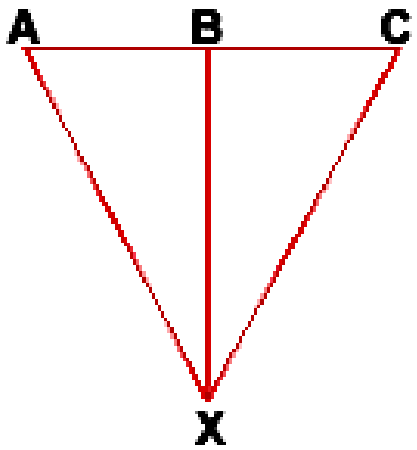
Since the earth appears convex from all parts, it must be spherical. Furthermore, every weighted body is carried naturally toward the center. If we concede that certain parts of the earth are further away from the center than others because of their size, it would be necessary that the small parts which encircle them be pressed, repelled and removed from the center, to the point where equality of distance and pressure being obtained, everything being constituted in equilibrium and repose like two wooden beams which mutually support each other or two athletes of the same force, mutually held in a clasp. If the different parts of the earth are equally far from the center, it is necessary that its form be spherical.

In addition, since the fall of heavy bodies is always and everywhere made towards the center, all converge towards the same point and each falls vertically, that is to say that angles which are always equal are made with the surface of the earth, then it must be concluded that the surface of the earth is spherical.

The surface of the sea and all tranquil waters are also spherical. This can be recognized in this manner If, situated on the shore, one observes an object from which one is separated by the sea, such as a hill, a tree, a tower, a ship, or the earth itself, then if one lowers one's gaze towards the surface of the water, one no longer sees anything, or one sees a smaller part of the object, the convexity of the surface of the sea masking the object. And often, while sailing, from the bridge of the ship neither earth nor any other vessel can be seen, but a sailor climbs up high on a mast pole and

can see them, being higher and so overcoming the convexity of the sea which was causing the obstacle.

It can be physically and mathematically demonstrated that the surface of every body of still water must be spherical in form. Water indeed tends to flow from higher towards lower levels. But the higher parts arc further away from the center of the earth, and the lower parts are less so. The surface of the water being presumed planar let us call ABC a straight line on this surface. From the center of the earth, such as point X. we draw the perpendicular to the base XB, and we draw the lines XA and XC to the extremities of this base. It is evident that the two lines XA and XC are both longer that XB, and that the two points A and C are further from the center than point B, and consequently, higher than point B. Water will flow therefore from points A and C towards the lower point B, until the latter point, surrounded with new water would be as far from point X as A and B are. Similarly, all points of the surface of the water will be at the same distance from X; therefore water presents the spherical form and the entire mass of the earth's water is spherical.



It cannot be said that the height of mountains or the depth of valleys would be contrary lo this thesis and prove that the earth is not an exact sphere. Eratosthenes indeed shows us that the circle of the earth, measured following the circumference of a great circle, has an approximate length of 252.000 stades, and Archimedes tells us that a circumference of a circle, developed as a straight line, has the value of three times and nearly a seventh of the circle's diameter. The diameter of the earth will then have the value of approximately 80,182 stades. Three times this number, plus a seventh of this number gives, in fact, 252,000 stades.

Now, according to Eratosthenes and Dicearchus, the vertical height of the highest mountains above the lowest plane is 10 stades. They have deduced this result from observations made with *the diopter* (a kind of graphometer) which allows the measurement of heights according to certain intervals. The height of the largest mountain would therefore be equal to an eight-thousandth part of the total diameter of the earth. If we were to make a sphere one foot (1 foot = 30,48 cm) in diameter, the width of one finger being about equal to $12^1/_2$ diameters of a grain of millet, the diameter of our sphere would equal 200 millet-grain diameters or a little less. Since the foot has the value of 16 fingers (finger – the smallest of the ancient Greek measures of length equal to 0.018 of the meter – or 0.916 of a yard), the finger has the value of 12 millet-grain diameters, and 16 times 12 is 192. One quarter of the diameter of a grain of millet is then larger than the eight-thousandth part of a foot, since 40 times 200 is 8,000.

But we have seen that the height of the largest mountain is nearly an eight-thousandth part of the diameter of the earth, thus the relationship of a quarter of the diameter of a grain of millet to this one-foot sphere is greater than the relationship of the height of the highest mountain to the diameter of the

earth. And the relationship of the sphere having a quarter of the thickness of a grain of millet to the sphere of a foot in diameter is greater than the relationship of the sphere of 10 stades in height to the height of the terrestrial sphere.

The sphere having a diameter of a quarter of the diameter of a grain of millet is the 64,000th part of a whole grain. The spherical mountain of 10 stades in diameter has the value of nearly 524 cubic stades, and the whole earth as a sphere, has the value, in cubic stades, 270 third myriads, 250 second myriads, 4350 first myriads, 8296 and the fraction $^{11}/_{21}$.

Furthermore, it is demonstrated that the rectangle formed by the diameter of a sphere and the circumference of a great circle, opened out as a straight line, equals 4 times the surface of a quarter of the sphere, and this quarter equals the area of the circle. The square of the diameter is to the area of the circle as 14 is to 11 (The ratio of 14 : 11 for finding the area of the circle is attributed to Archimedes, but can be derived as well from the approximately 7 : 11 ratio of the vertical half-section of the Gizeh pyramid.); for the circumference of the circle equals three times the diameter plus a seventh part of this diameter. If the diameter is seven, the circumference is 22. A quarter of this circumference is 5 + $^{1}/_{2}$. Therefore, the square of the diameter being 49, the circle having this diameter is 38 + $^{1}/_{2}$; and if we double this in order to make the $^{1}/_{2}$ disappear, the square of the diameter being 98, the circle having this diameter will be 77. However the relationship of these numbers, expressed in the smallest prime terms, is that of 14 lo 11, since the largest common measure of these numbers is 7, which is contained one and a half limes in the cylinder, according to Archimedes, is also equal to the relationship of 14 to 11.
Thus, then, when the cube of the diameter of the circle equals 14, the circumscribed cylinder will equal 11 and the sphere 7 + $^{1}/_{3}$.

It is thus that one finds volumes expressed in numbers of the terrestrial sphere and of the highest mountain, if a mountain 10 stades high were a sphere, it would be much smaller, with respect lo the earth, than $^{1}/_{64,000}{}^{th}$ of a grain of millet in relation to a sphere of one foot in diameter. But mountains are not spherical, and, as we see them, they are much smaller. But one such part of a grain of millet, whether it be superimposed on a sphere of one foot in diameter, or whether it be picked up and placed in a hollow, will not produce any difference in form. The highest mountains of 10 stades have the same relationship with the earth, they will not therefore prevent the whole of earth and sea from truly being a sphere.

The encircling of the earth therefore is valued. 252.000 stades.
the diameter ..... ……..80,182 stades.
the square of the diameter…….6,429,153,124 square stades,
the cube ..... …… 515.502,355.788.568 cubic stades,
a fourteenth of this cube .. ……...36,821.596.842,040 and $^{4}/_{7}{}^{ths}$.

The product of this number by $^{22}/_{3}$ is equal to the volume of the earth which therefore has the value in cubic stades of 270.025,043,508,297 and $^{11}/_{21}$.

The earth is spherical and placed at the center of the world. If it were removed from this position it would not have half the sky above it and half below. Furthermore, straight lines drawn from any point at the extremities of the celestial sphere would not be equal. That the volume of the earth has no perceptible relationship with the expanse of the universe, that it occupies but a point in this universe, is shown by the points of the sundials in every inhabited place on earth. They can in fact be taken for the center of the solar orbit, for in changing location, one cannot observe any perceptible parallax. If therefore there is necessarily a center for the ensemble of all spheres, all the points on earth appear to be this center. It is then evident that the whole earth is only a point with respect to the entire sphere of the sun and even more so with respect to the sphere of the stars, it is for this reason that half of the world, or nearly so, is always apparent to our eyes.

Although we might say many other things about the form of the universe and of the earth, and of earth's central position as well as its size, and the unknown size of its relation to the universe, what has been demonstrated by Adrastus in the manner shown above will be sufficient for the exposition of what follows. Here is what he next says:

ON THE CELESTIAL CIRCLES

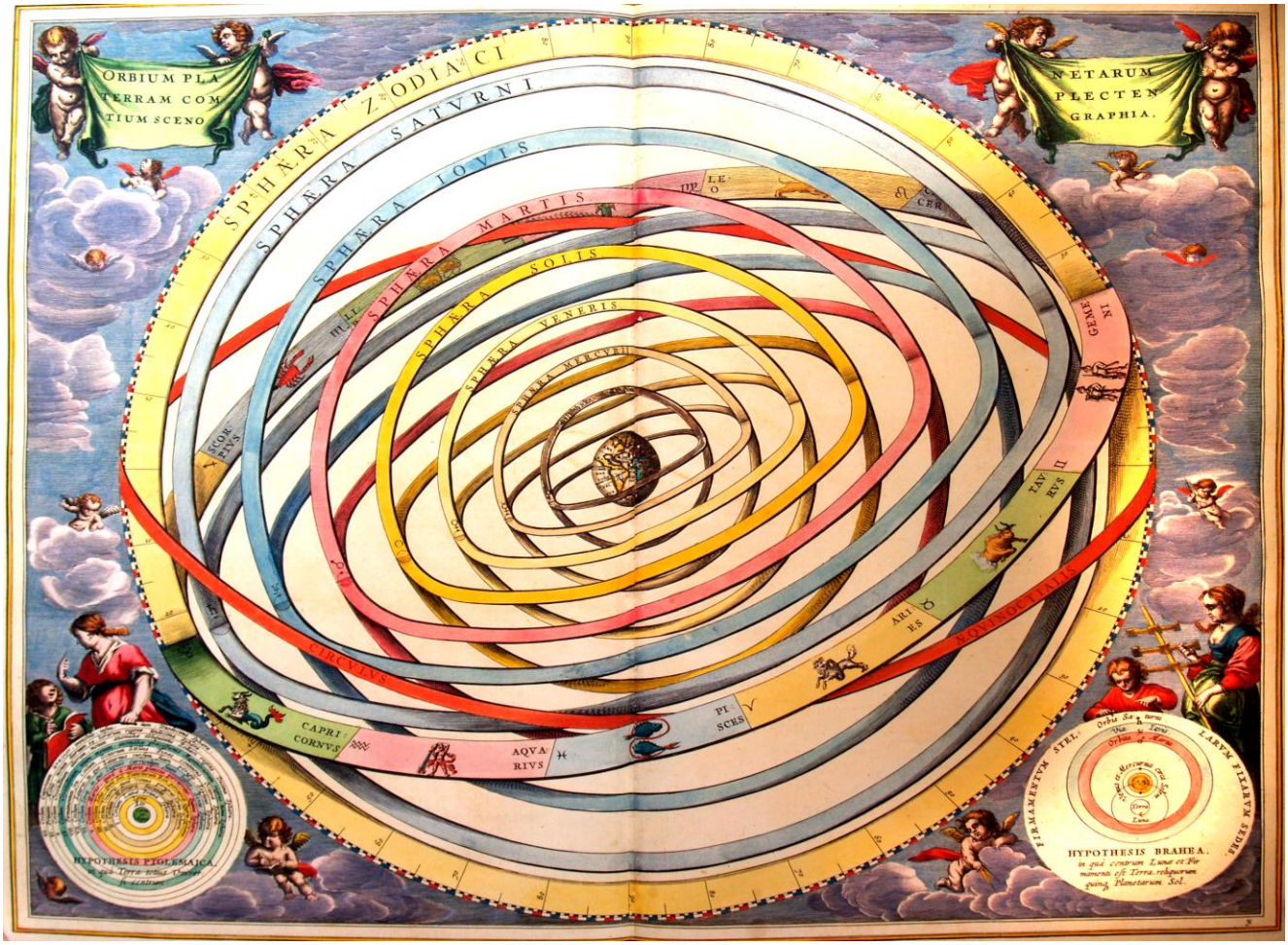
The celestial sphere turning around its immobile poles and the axis which joins them, at the middle of which is fixed the earth, and all the stars carried by this sphere and all the points of the sky, describe parallel circles, that is to say circles everywhere equidistant, perpendicular to the axis, and drawn from the poles of the universe as centers. One can count the circles described by the stars, but the circles described by the other points are innumerable. Some of these circles have been given particular names which it is useful to know in order to take into account what happens in the heavens.

There is one above us, around the always apparent and visible pole. It is called the arctic circle, because of the constellations of the Bears which it crosses. Another, on the opposite side, equal to the first, circles the pole which we never see, is itself always invisible to us, and is called the antarctic circle. The one in the middle, which is a large circle, divides the whole sphere into two equal parts and is called the equator, because on these corresponding regions of the earth, there is an equality between the days and nights, for other places in which one sees the sun rise and set according to the general movement of the universe, the durations of the day and night are equal when the sun describes this circle.

Between the equatorial circle and the two arctic circles there are the two tropics, the tropic of summer, situated for us on this side of the equatorial circle, and on the other side, the tropic of winter. The sun. in its revolution, sometimes moves near to one and sometimes lo the other. The zodiac is in fact obliquely extended between these two circles.

The zodiac is also a great circle. It touches each tropic at one point: the tropic of summer at a point in Cancer and the other tropic at a point in Capricorn. It cuts the equatorial circle into two equal parts and is itself also divided by this circle at a point in Aries and it a point in Taurus. It is in this zone that the sun, moon, and other planets move: Saturn, Jupiter, Mars, Venus, or also Lucifer, Mercury.

The circle which is the boundary of our vision and divides the sky as a whole into two equal parts — the earth being the obstacle from our viewpoint — is called the horizon. Of these parts the one above the earth is the visible hemisphere, and the other below is the invisible hemisphere. As it is also a great circle of the sphere, it also cuts the great circles such as the equator and the zodiac into two equal parts. If two stars are diametrically opposed, when one risen, the other sets. The horizon divides the meridian into two equal parts.



For there is another great circle, called the meridian, which passes through the two poles of the world, and which is conceived as perpendicular to the horizon. It is called meridian because the sun cuts it at the middle of the day, being at the highest point of its course above the horizon. It is sometimes called "truncated" (Colure, de la Κολούρος, truncated), because one of its parts, that which is on the side of the invisible pole, is hidden from us.

The equatorial (circle) and the two tropics situated on either side of it are circles which are given and fixed in size and position. It is said that points and lines are 'given' in position when they always occupy the same place; it is said that surfaces, lines and angles are given in size when equal sizes can be found. Now the equatorial circle and the two tropics placed on cither side of it always have the same position, they are always fixed, and one could find equal circles: the zodiac, the horizon and the meridian being equal to the equatorial circle, and the tropic of summer being equal to the tropic of winter and reciprocally. This is why they are always given, it is not in our power to render them as this or that they are naturally so, they are given, we cannot determine them.

But those which it is in our power to render as this or that are not naturally given Those which are naturally given, that is, which are fixed and exist by themselves, are the equatorial circle and the circles situated on either side of it, given in size and position. The zodiac is a circle given in size and position with respect to the sky, but in relation to us, it is not given in position. Indeed, for us it is not fixed, because of its obliquity in the universe, which to us shows it changing in place.

The meridian and the horizon are also given in size, for they are the great circles of the celestial sphere, but they change in position according to the earth zones and are different in different locations

on the earth. In fact we do not all have the same horizon nor the same zenith, nor the same meridian. As for the arctic and the antarctic circles which are neighbors of the poles, they are not given in either size or position (One would call the arctic circle in each location the parallel limit of the stars always visible in that place, and the antarctic circle the parallel limit of the always invisible stars), but according to the differences in more northerly or southerly zones, they are seen to be larger or said that the sphere is right because in this region of the earth all the parallel circles are perpendicular to the horizon.

Each of the other circles is a true circle terminated by a single line, but the one called the zodiac shows a certain breadth, like the cylinder of a drum. The animal figures are imagined on the inside of this cylinder. The circle in the middle of these signs is called the great circle which touches the two tropics at one point on each of them, and cuts the equatorial circle into two equal parts. The two circles which define the width of the zodiac on either side are smaller circles.

THE STARS

Most of the stars are fixed: they are carried together by a unique and simple circular movement, with the first sphere which is the largest, as if they were fixed to it and as if they were moved by it. They always have the same relative position on the sphere, and maintain the same order between each other and do not experience any change of form or movement, nor of size or color.

THE PLANETS

The sun, moon and other stars which arc called errant are carried with the universe in the diurnal movement from east to west, just like the fixed stars. But apart from this movement, each day they appear to have several other complicated motions. For, through a movement of their own, they go toward the zodiac signs which follow them (in the diurnal movement) and not to the zodiac signs which precede them, carried along in the contrary direction of the universe in a course which is called movement in longitude. In addition, they have a movement in latitude, from north to south and reciprocally, while accomplishing their course in the contrary direction to the movement of the universe. Attentive observers see them moved from the tropic of summer to the tropic of winter and reciprocally, through the obliquity of the zodiac.

And within the breadth of the zodiac, they are sometimes seen further north from the middle circle, and sometimes further south; some are lower down, others less so. Besides, they vary in size, sometimes being further distant, and sometimes being closer to the earth in the depths of space. It is for this reason that the speed of their movement through the zodiacal signs appears unequal. They do not cover the same distance in space in the same amount of time, they go faster when they appear larger because of their lesser distance from earth, and they go less fast when they appear smaller because of their greater distance.

The distance covered on the zodiac is slight for the sun, since it is just about one degree out of 360 For the moon, as the ancient astronomers have said, and for Venus, it is larger, since it is about 12 degrees. Hermes covers about 8, Mars and Jupiter about 5, and Saturn nearly 3. The moon and the sun each appear to deviate equally in latitude from the circle of the middle of the zodiac The other planets do not deviate equally from it, but are more northward in one sign, and more southward in another.

As for the length of the circle of the zodiac, from one fixed point to this same point, the moon, going towards the same zodiacal signs and not towards the preceding ones, travels it in $27^{1}/_{3}$ days, the sun

in a year having the approximate value of 365 $^{1}/_{4}$ days. Venus and Hermes travel in an unequal movement, but with little difference in duration, and it can be said that they have the same speed as the sun, since they are always seen beside it, sometimes following it, sometimes preceding it. Mars achieves its course in a little less than 2 years, Jupiter in about 12 years, and Saturn in a little less than 30 years.

The conjunctions with the sun, the appearances and disappearances which are called the risings and the settings, are not the same for all the planets. The moon, in fact, after its conjunction with the sun, having a more rapid movement than the sun towards the zodiacal signs which follow, appears first and rises in the evening, while it disappears and sets in the morning. Inversely, Saturn, Jupiter and Mars, which arrive less quickly than the sun at the following signs, are preceded and overtaken by it, that is to say, that these planets always set in the evening and rise in the morning (after the conjunction).

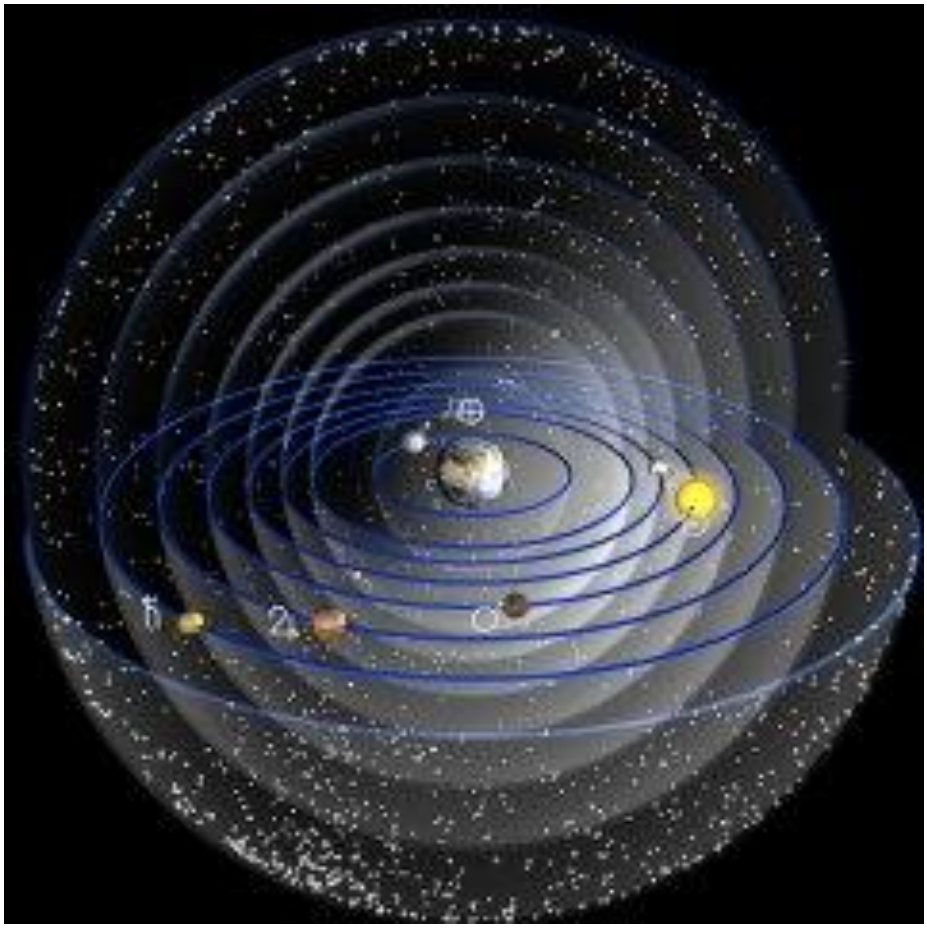
Venus and Hermes, which have a movement equal to that of the sun, always appear near to it; sometimes these two stars follow it, sometimes they precede it: sometimes they appear in the evening and disappear also in the evening, sometimes they appear at the curliest dawn and disappear with the day. While the other planets are far from the sun, at every interval, up to the point where they are diametrically opposite, these two stars are on the contrary always seen near the sun. Hermes is about 20 degrees away from it, that is, about two thirds of a zodiacal sign, either to the east or to the west. Venus is about 50 degrees to the east or to the west.

Rising means several things. First, properly and commonly, for the sun and the other stars, by their elevation above the horizon; next, for their shining to begin to distinguish itself from the rays of the sun, which is still properly a manner of rising. There still remains the rising called rising at "night-edge" ("Night-edge" here applies to the *early beginning* and *early end* of the night, i.e. at early dawn and early sunset. – Toulis), which is produced in the east after the setting of the sun, in the part of the sky diametrically opposite It is called "night-edge" because it occurs at the edge of the night, i.e. at its beginning. Similarly the first setting is the descent below the horizon. Next is the setting produced by the diffusion of the brilliance of the star by the luminous rays of the sun; which is properly called a disappearance. There still remains the setting called again night-edge, but at dawn, when the sun rises, a star disappears in the part of the horizon diametrically opposite.

Among the risings and the settings depending on the sun and its rays, that is to say among the phenomena of appearance and disappearance, some occur in the morning, others in the evening. The rising of the star belongs to the morning when the star preceeding the rays of the sun appears before it in the east, as with the rising of Canis Major. The rising belongs to the evening when the star begins to appear after the setting of the sun, as we have said of the new moon. Similarly, the setting belongs to the morning when the star, which in the preceding days was rising before the sun, like the moon, ceases to appear at its approach; the setting is of the evening when the sun, being very close to a star in the west, causes that star to become invisible because of the radiance of its neighbor.

THE MYTH OF PAMPHYLION IN PLATO'S REPUBLIC

Plato, at the end of *the Republic*, wishing to exhort justice and virtue, recounts a fable in which, speaking of the arrangement of the celestial bodies, he says that an axis traverses the celestial pole like a pillar. He adds that there is another spine-like axis with hollow vertebra nested one next to the other. These vertebra are none other than the spheres carrying the seven planets. The last sphere being that of the stars, envelopes all the others. He shows the order of these spheres with respect to the distance of each of the stars, to their color and to the speed of their movement in the opposite direction to that of the universe. This is what he says:



"…This is how it was made: in its form it resembled the spindle-wheels of our world, but according to the description given by Pamphylian, it should be represented as containing in its hollow another smaller spindle-wheel, which itself received a third, tike large vessels fitted one inside the other. There was thus a third, a fourth and yet four more of them. There were, therefore, eight spindle-wheels in all, placed one within the other. Their circular rims could be seen from above, and they presented the continuous curved surface of a single spindle around the shaft passing through the center of the first. The circular rim of this exterior spindle was the widest then that of the sixth, the fourth, the eighth, the seventh, the fifth, the third and of the second, diminishing in width in that order.

The rim of the largest spindle (the sphere of the stars) was of different colors, the rim of the seventh (sphere of the sun) was of a very brilliant color, that of the eighth (sphere of the moon) borrowed its color and its brilliancy from the seventh. The color of the circles of the seventh and the fifth (Saturn and Hermes) was nearly the same and they were more yellow than the others: the third (Jupiter) had a very white color, that of the fourth (Mars) was slightly red. Finally the sixth (Venus) occupied second place in brilliancy and whiteness.

The complete exterior spindle made its revolution in the same direction as the universe, and, in the interior, the seven concentric spindles moved slowly in the opposite direction. The movement of the eighth was the most rapid; the movements of the seventh, sixth and fifth were less of an equal speed; the fourth which has a more rapid retrograde movement than the other spindles is third in speed, as it appeared to them; the third had only the fourth speed and the second had only the fifth."

This is what Plato says. We explain this passage in the *Commentaries on the Republic*. Also, we have constructed a sphere according to his explanations. Plato says, in fact, that it would be useless work to wish to expose these phenomena without the images which speak to the eyes…

The same demonstrations can be applied to the other planets. The sun appears to make all these movements, in both hypotheses, with regularity, for the times or its return to the same longitude, to the same latitude, and to the same distance which products the irregularity called anomaly, are so little different from each other that most mathematicians regard them as equal 365 ¼ days. Thus, when one attentively considers the time of the return in longitude during which the sun travels the zodiac, going from one point back to the same point, from one solstice to the same solstice, or from one equinox to the same equinox, it is very close to the time noted above, so that at the end of four years, the return to a point at the same longitude occurs at the same hour.

As for the time of the anomaly after which the sun, at the point furthest from the earth, appears smallest and slowest in its movement towards the following zodiac signs, or after which, at the point closest to the earth, it appears to have the largest diameter and the greatest speed, it is close to 365 ½ days, so that at the end of two years the sun appears to return to the same distance at the same hour. Finally, the time of its return in latitude, the time after which, tuning from the extreme north or south point, it returns to the same point in such a way as to give the same shadow-lengths on the sundials, is $365^{1}/_{8}$ days. Consequently, it might be said that at the end of eight years, it will return at the same hour to the same point of latitude.

Regarding each of the other planets, we have said that their various times vary greatly, some are longer, some are shorter. The durations of the returns appears so much the more variable and changing in one hypothesis as in the next, that it is not in the same lapse of time that each planet travels its epicycle and the epicycle its concentric circle (in the zodiac): the movements are more rapid for some and slower for others by reason of the inequality of the circles, of the inequality of the distances from the center of the universe and of the differences of obliquity with respect to the circle of the middle of the zodiac signs, that is to say, of the differences of inclination and of position.

As a result, it happens that for all the planets, the stations and the returns, whether towards the preceding zodiac signs or the following zodiac signs, do not occur in a similar manner. One observes the phenomenon for the five planets, but in a manner which is not absolutely similar. For the sun and the moon, it does not occur at all; indeed these two never appear to advance, nor to remain stationary, nor to retrogress…